Kalotay Analytics

A DIVISION OF ANDREW KALOTAY ASSOCIATES, INC.

Illustrative Refunding Analysis
According to Massachusetts State Treasury Guidelines

Prepared on behalf of:

April 19, 2013

Market Inputs

- ✓ New issue scale (YTC or YTM) for the Commonwealth of Massachusetts
  - Show coupon by maturity; indicate call provision
  - E.g. 5% for all maturities, NC-10 at par
- ✓ Treasury, SLGS, or Agency rates used in advance refundings

Throughout this document:

✓ indicates mandatory inclusion with refunding proposals

Convert Market Scale Into Format for Standard Bond Analytics*

Use 15% interest rate volatility to calculate:

- ✓ Optionless (non-call life) par yield curve
- ✓ Spot (pure discount) rates

Note: Mass. may change vol. specification from time to time, depending on market conditions

**Such as would be used on the Bloomberg OAS1 page, for example*

Scale Conversion Example

Commonwealth of Massachusetts
General Obligation Bonds
Indicative Interest Rates as of April 2, 2013

			Premium Coupons 10 year par call		
	MMD	Maturity	Coupon	Spread to MMD	Yield
1	0.20%	2014	5.000%	0.00	0.20%
2	0.31%	2015	5.000%	3.00	0.34%
3	0.47%	2016	5.000%	6.00	0.53%
9	1.71%	2	5.000%	22.00	1.93%
10	1.89%	3	5.000%	23.00	2.12%
11	2.02%	4	5.000%	25.00	2.27%
14	2.39%	7	5.000%	25.00	2.64%
15	2.48%	8	5.000%	25.00	2.73%
16	2.54%	9	5.000%	25.00	2.79%
28	3.07%	21	5.000%		3.07%
29	3.08%	22	5.000%		3.08%
30	3.09%	23	5.000%	22.00	3.31%

15% vol

iteRate - 1.0

Maturity (yrs)	Par NCL Rate	YTC	YTM
1.00	0.200	0.200	0.200
2.00	0.344	0.340	0.340
3.00	0.541	0.530	0.530
9.00	2.016	1.930	1.930
10.00	2.213	2.120	2.120
11.00	2.540	2.270	2.464
30.00	3.696	3.310	4.162

Conversion methodology described in:

What Makes the Muni Yield Curve Rise?

Journal of Fixed Income (Winter 2009)

Treasury, SLGS, or Agency NCL Rates

Time	Rate
0.083	0.090%
0.25	0.093%
0.5	0.094%
1	0.165%
2	0.237%
3	0.342%
5	0.751%
7	1.223%
10	1.836%
20	2.779%
30	3.075%

Provide at least up to the longest relevant call date

Transaction Costs

- ✓ Current underwriting fee
  - Affects savings in the contemplated transaction
- ✓ Schedule of fees by maturity
  - Needed to capture costs in future refundings
    - Reduces option value

Underwriting Fees Example

AccruedAndFeeSource	NewIssue
Fee	0.5%
PVDate	4/2/2013

Current fee reduces savings
(Show for each bond)

Applicable to future refunding opportunities; reduces option value of refunding candidate and of callable replacement bond

Maturity	Underwriting Fees
1	0.1%
1.5	0.15%
2	0.2%
3	0.25%
4	0.3%
5	0.35%
7	0.4%
10	0.45%
15	0.5%
20	0.55%
30	0.6%

Analytical Results

Outstanding bond

- ✓ PV of cash flows (to nominal maturity)
- ✓ Option value (call, advance refunding, total)

Replacement bond

- ✓ PV of cash flows (to nominal maturity)
- ✓ Option value (if relevant)

Refunding Efficiency

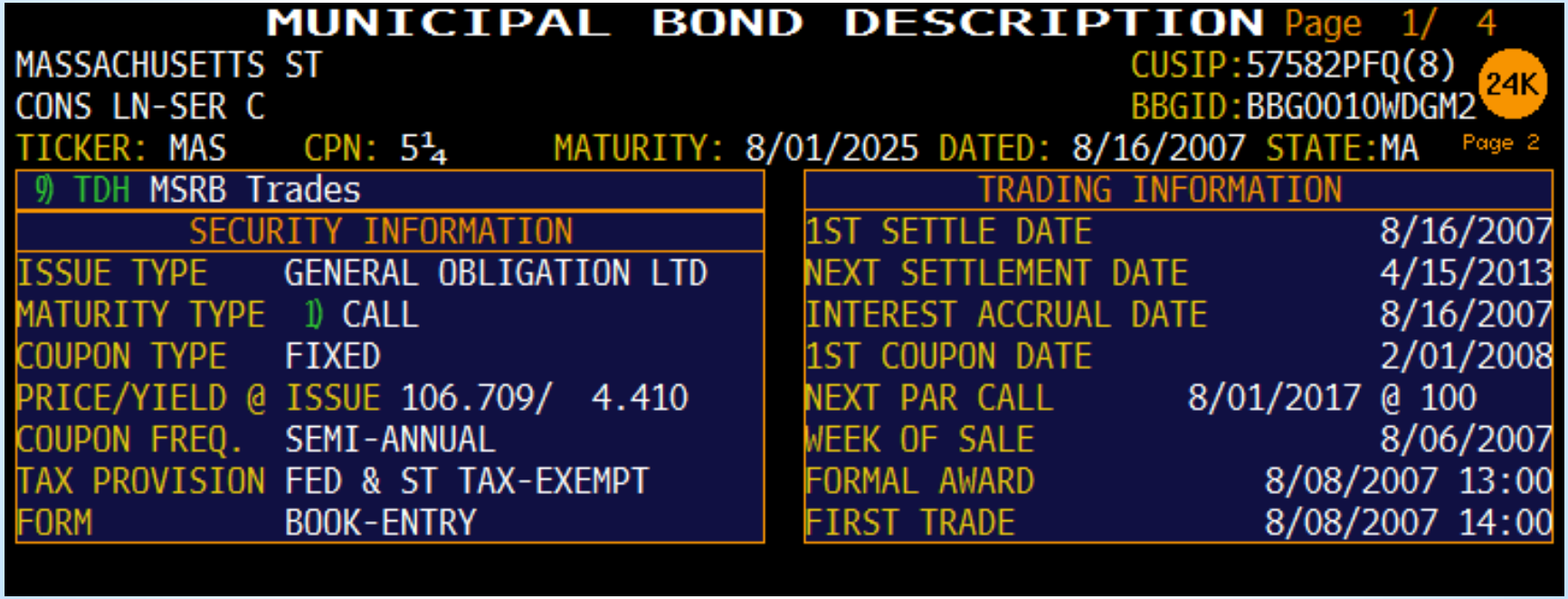
Use Generalized Refunding Efficiency Formula

$$Efficiency_{generalized} = \frac{PV\ Savings}{Loss\ of\ Option\ Value}$$

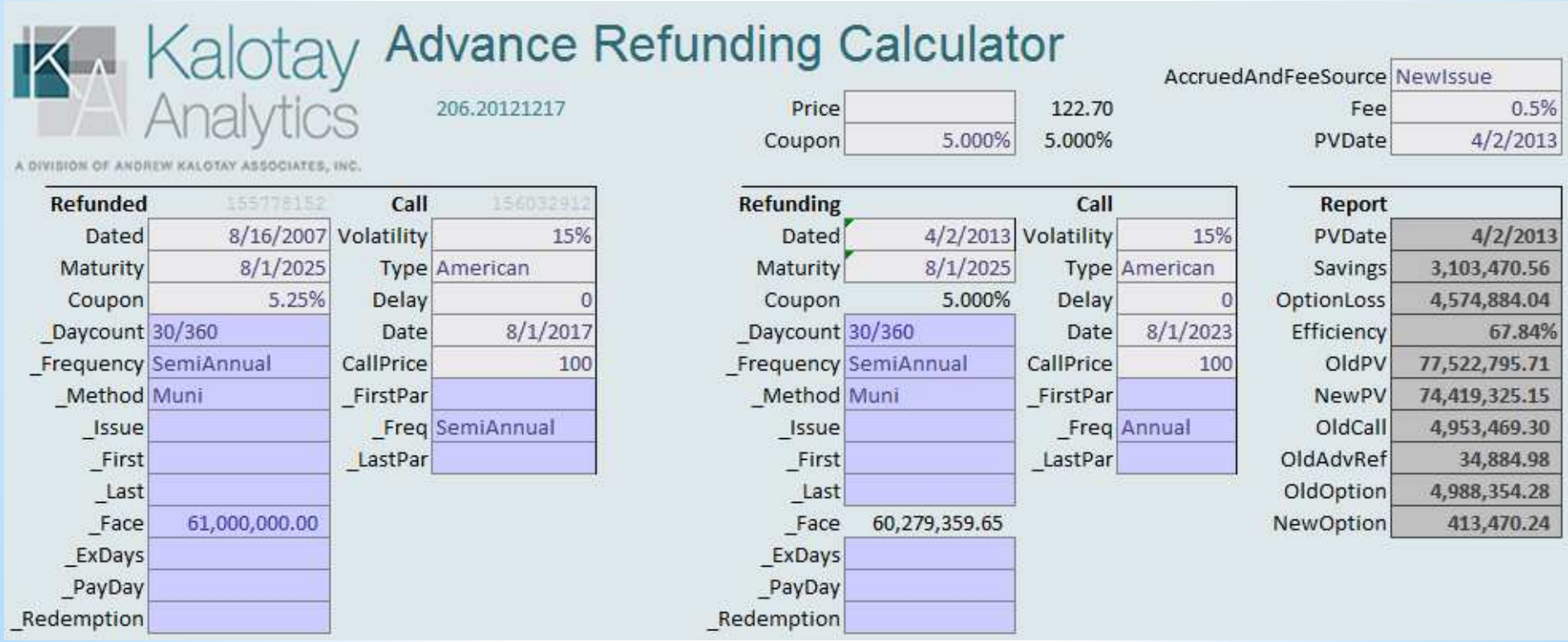
Present only those candidates with efficiency of at least 50%

Refunding Efficiency: A Generalized Approach
Applied Financial Economic Letters, 2007
Efficiency when the refunding bond is also callable

Refunding Candidate



Refunding Efficiency Example



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Advance Refunding Calculator

206.20121217

Price		122.70
Coupon	5.000%	5.000%

AccruedAndFeeSource	NewIssue
Fee	0.5%
PVDate	4/2/2013

Refunded		Call	
Dated	8/16/2007	Volatility	15%
Maturity	8/1/2025	Type	American
Coupon	5.25%	Delay	0
_Daycount	30/360	Date	8/1/2017
_Frequency	SemiAnnual	CallPrice	100
_Method	Muni	_FirstPar	
_Issue		_Freq	SemiAnnual
_First		_LastPar	
_Last			
_Face	61,000,000.00		
_ExDays			
_PayDay			
_Redemption			

Refunding		Call	
Dated	4/2/2013	Volatility	15%
Maturity	8/1/2025	Type	American
Coupon	5.000%	Delay	0
_Daycount	30/360	Date	8/1/2023
_Frequency	SemiAnnual	CallPrice	100
_Method	Muni	_FirstPar	
_Issue		_Freq	Annual
_First		_LastPar	
_Last			
_Face	60,279,359.65		
_ExDays			
_PayDay			
_Redemption			

Report	
PVDate	4/2/2013
Savings	3,103,470.56
OptionLoss	4,574,884.04
Efficiency	67.84%
OldPV	77,522,795.71
NewPV	74,419,325.15
OldCall	4,953,469.30
OldAdvRef	34,884.98
OldOption	4,988,354.28
NewOption	413,470.24

Cash Flow Savings Example

		Refunded					Refunding		Net Savings	
Date	Interest	Principal	PV	Spot	Discount	Interest	Principal	PV	CashFlow	PV
4/2/2013							0	0	0	0
8/1/2013	1,601,250	0	1,600,192	0.200%	0.99934	996,281	0	995,623	604,969	604,570
2/1/2014	1,601,250	0	1,598,594	0.200%	0.99834	1,506,984	0	1,504,484	94,266	94,110
8/1/2014	1,601,250	0	1,595,954	0.249%	0.99669	1,506,984	0	1,502,000	94,266	93,954
2/1/2015	1,601,250	0	1,591,879	0.321%	0.99415	1,506,984	0	1,498,165	94,266	93,714
8/1/2015	1,601,250	0	1,586,007	0.411%	0.99048	1,506,984	0	1,492,638	94,266	93,369
2/1/2016	1,601,250	0	1,578,341	0.510%	0.98569	1,506,984	0	1,485,423	94,266	92,917
8/1/2016	1,601,250	0	1,569,566	0.601%	0.98021	1,506,984	0	1,477,165	94,266	92,401
2/1/2017	1,601,250	0	1,559,848	0.685%	0.97414	1,506,984	0	1,468,019	94,266	91,829
8/1/2017	1,601,250	0	1,546,335	0.807%	0.96570	1,506,984	0	1,455,302	94,266	91,033
2/1/2018	1,601,250	0	1,529,762	0.948%	0.95535	1,506,984	0	1,439,704	94,266	90,057
8/1/2021	1,601,250	0	1,366,257	1.914%	0.85324	1,506,984	0	1,285,825	94,266	80,432
2/1/2022	1,601,250	0	1,337,787	2.046%	0.83546	1,506,984	0	1,259,031	94,266	78,756
8/1/2022	1,601,250	0	1,310,079	2.163%	0.81816	1,506,984	0	1,232,954	94,266	77,125
2/1/2023	1,601,250	0	1,282,327	2.272%	0.80083	1,506,984	0	1,206,836	94,266	75,491
8/1/2023	1,601,250	0	1,246,999	2.435%	0.77877	1,506,984	0	1,173,588	94,266	73,411
2/1/2024	1,601,250	0	1,207,062	2.626%	0.75383	1,506,984	0	1,136,002	94,266	71,060
8/1/2024	1,601,250	0	1,170,043	2.788%	0.73071	1,506,984	0	1,101,162	94,266	68,881
2/1/2025	1,601,250	0	1,134,082	2.937%	0.70825	1,506,984	0	1,067,319	94,266	66,764
8/1/2025	1,601,250	61,000,000	42,977,153	3.074%	0.68652	1,506,984	60,279,360	42,417,702	814,906	559,451

Spot rates derived from input scale

Note That …

- At a 15% interest rate volatility, the implied 30-year NCL rate is 3.696% (Scale Conversion Example)
- The spot rate for discounting a 2/1/2025 cash flow is 3.074% (Cash Flow Savings Example)
- According to the Refunding Efficiency Example,

  PV savings=$3.103MM

  Option value of the outstanding 5.25% bonds=$4.988MM, ($4.953MM call, $0.035MM advance refunding)

  Option value of the refunding bond=$0.413MM

  Net loss of option value=$4.575MM

  Refunding efficiency=67.85%

Comments on Analytics

- Results calculated using Kalotay's iteRate™ and Advance Refunding Calculator™
  - However, proposals may utilize any suitable software
- Discounting and call option valuation should follow standard 'textbook' methodology
  - See "Valuation of Municipal Bonds with Embedded Options"
    Handbook of Municipal Securities, 2008 , ed. F. Fabozzi
- Value of the advance refunding option was calculated using Kalotay's proprietary algorithm
  - Reasonable alternatives are acceptable
  - See "The Timing of Advance Refunding of Tax-Exempt Municipal Bonds" – Kalotay and May
    Municipal Finance Journal (Fall 1998)

Contact Information

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The Commonwealth of Massachusetts
General Obligation Bonds
Refunding Analysis

Rates as of	[add date]
Delivery Date	5/1/2013
Costs of Issuance	$5.00
Bonds Ranked by:	Refunding efficiency, high to low

Refunding Efficiency
Volatility Assumption 15%
Interest Rate Model Black-Karasinski

			Existing Bond								New Bond				Negative Arbitrage		
	CUSIP	Series	Coupon	Maturity	Par	Call Date	Call Price	Yrs. to Call	Yrs. to Maturity	Call to Maturity Spread	Coupon	Current Market Yield	Price	Maturity	Escrow Yield	$ Negative Arb	% of Par
1	57582N7G4	Consolidated Loan of 2006, Series D	5.000%	8/1/2019	15,990,000	8/1/2016	100.00	3.25	6.25	3.00	2.000%	1.260%			0.400%	(474,362)	-2.97%
2	57582N7H2	Consolidated Loan of 2006, Series D	4.250%	8/1/2020	6,620,000	8/1/2016	100.00	3.25	7.26	4.00	2.000%	1.520%			0.400%	(251,252)	-3.80%
3	57582N7K5	Consolidated Loan of 2006, Series D	4.300%	8/1/2021	2,910,000	8/1/2016	100.00	3.25	8.26	5.00	2.000%	1.760%			0.400%	(133,648)	-4.59%
4	57582PFH8	Consolidated Loan of 2007, Series C	5.000%	8/1/2019	31,000,000	8/1/2017	100.00	4.25	6.25	2.00	2.000%	1.260%			0.620%	(903,571)	-2.91%
5	57582PKK1	Consolidated Loan of 2007, Series C	5.250%	8/1/2021	34,000,000	8/1/2017	100.00	4.25	8.26	4.00	2.000%	1.760%			0.620%	(1,754,794)	-5.16%
6	57582PFM7	Consolidated Loan of 2007, Series C	5.250%	8/1/2022	53,000,000	8/1/2017	100.00	4.25	9.26	5.00	2.000%	1.960%			0.620%	(3,200,822)	-6.04%
7	57582PFJ4	Consolidated Loan of 2007, Series C	5.000%	8/1/2020	32,000,000	8/1/2017	100.00	4.25	7.26	3.00	2.000%	1.520%			0.620%	(1,303,942)	-4.07%
8	57582PFN5	Consolidated Loan of 2007, Series C	5.250%	8/1/2023	58,000,000	8/1/2017	100.00	4.25	10.26	6.00	2.250%	2.150%			0.620%	(3,982,602)	-6.87%

Escrow Efficiency	PV Savings		Break-Even Analysis		Option Value (% old par)			Refunding Efficiency	Cumulative Results		
										PV Savings	
	$ Savings	% of Par	Break-Even Rate Increase to Eliminate Savings	PV01	Old Bond	New Bond	Net Loss		Refunded Par	$ Savings	% of Par
70.2%	1,117,273	6.99%	0.25%					99%	15,990,000	1,117,273	6.99%
60.0%	376,211	5.68%	0.26%					98%	22,610,000	1,493,484	6.61%
57.7%	182,515	6.27%	0.24%					97%	25,520,000	1,675,999	6.57%
54.3%	1,074,062	3.46%	0.22%					96%	56,520,000	2,750,061	4.87%
56.5%	2,275,123	6.69%						95%	90,520,000	5,025,184	5.55%
56.1%	4,086,293	7.71%						94%	143,520,000	9,111,477	6.35%
54.4%	1,554,512	4.86%						93%	175,520,000	10,665,989	6.08%
55.0%	4,861,211	8.38%						92%	233,520,000	15,527,200	6.65%

The Commonwealth of Massachusetts
General Obligation Bonds
Refunding Analysis

Mass. G.O. Scale for Refunding Bonds	
Maturity	Rate
1	0.00%
2	0.00%
3	0.00%
4	0.00%
5	0.00%
6	0.00%
7	0.00%
8	0.00%
9	0.00%
10	0.00%
11	0.00%
12	0.00%
13	0.00%
14	0.00%
15	0.00%
16	0.00%
17	0.00%
18	0.00%
19	0.00%
20	0.00%
21	0.00%
22	0.00%
23	0.00%
24	0.00%
25	0.00%
26	0.00%
27	0.00%
28	0.00%
29	0.00%
30	0.00%

Mass. G.O. Spot Rate Curve	
Maturity	Rate
1	0.00%
2	0.00%
3	0.00%
4	0.00%
5	0.00%
6	0.00%
7	0.00%
8	0.00%
9	0.00%
10	0.00%
11	0.00%
12	0.00%
13	0.00%
14	0.00%
15	0.00%
16	0.00%
17	0.00%
18	0.00%
19	0.00%
20	0.00%
21	0.00%
22	0.00%
23	0.00%
24	0.00%
25	0.00%
26	0.00%
27	0.00%
28	0.00%
29	0.00%
30	0.00%

The Commonwealth of Massachusetts
General Obligation Bonds
Refunding Analysis

Summary of Refunding Results

	Current Market Rates	Scenario Analysis: 25 bps Decrease	Scenario Analysis: 25 bps Increase
Par Amount Refunded	$ -	$ -	$ -
Refunding Par Amount	$ -	$ -	$ -
Average Life			
Arbitrage Yield			
All-In TIC			
Escrow Yield			
Gross Savings ($)	$ 18,000,000	$ -	$ -
PV Savings ($)	$ -	$ -	$ -
PV Savings (%)			
Weighted Average Efficiency	95%	96%	94%
Negative Arbitrage	$ -	$ -	$ -

Principal Amortization
(current market rates)

Maturity	Par
8/1/2013	$ -
8/1/2014	10,000,000
8/1/2015	12,000,000
8/1/2016	13,000,000
8/1/2017	14,000,000
8/1/2018	15,000,000
8/1/2019	16,000,000
8/1/2020	17,000,000
8/1/2021	18,000,000
8/1/2022	19,000,000
Total	$ 134,000,000

Annual Budgetary Savings
(current market rates)

Fiscal Year	Gross Savings
2014	$ -
2015	2,000,000
2016	2,500,000
2017	1,000,000
2018	3,000,000
2019	2,000,000
2020	3,500,000
2021	250,000
2022	1,750,000
2023	2,000,000
Total	$ 18,000,000